Modelling and simulation of LQR and LFSV controllers in the Magnetic Levitation System (MLS)

Modelado y simulación de controladores LQR y RLVE al Sistema de Levitación Magnética (SLM)

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RESUMEN

In this article a control analysis in state variables is presented, applied to the nonlinear Magnetic Levitation System (MLS), which consists in keeping objects suspended in the air without any mechanical contact through the interaction of magnetic force. The design of Linear Quadratic Regulator (LQR) and Linear Feedback in State Variables (LFSV) controllers is implemented with the aim of comparing the results which guarantee a better stability performance in the system. The mathematical representation of the nonlinear and linearized model of the MLS plant is examined through the design of algorithms and simulation in Simulink-Matlad. In this way, the behavior of the system when there are perturbations and input changes is contrasted, with the priority of exerting a low control action as parameter of the system to be optimized.

Key words: Linearization; Stability; State observer; Controllers LQR and LSFV.

ABSTRACT

En este artículo se presenta un análisis de control de variables de estado, aplicada al Sistema no Lineal de Levitación Magnética (SLM), que consiste en mantener objetos suspendidos en el aire sin ningún tipo de contacto mecánico, a través de la interacción de la fuerza magnética. El diseño de los controladores Regulador Lineal Cuadrático, por sus siglas en inglés (LQR) y la Realimentación Lineal en Variables de Estado (RLVE), permiten modelar y simular la dinámica de control, con el objetivo de comparar los resultados que garantizan un mejor funcionamiento de la estabilidad en el sistema. La representación matemática del modelo no lineal y linealizado de la planta SLM se examina mediante el diseño de algoritmos y simulación en Simulink-Matlad. De esta manera, se contrasta el comportamiento del sistema ante perturbaciones y cambios en las variables de entrada, con la prioridad de ejercer una mínima acción de control como parámetro del sistema a optimizar.

Palabras clave: Linealización; Estabilidad; Observador de estado; Controladores LQR y RLVE.

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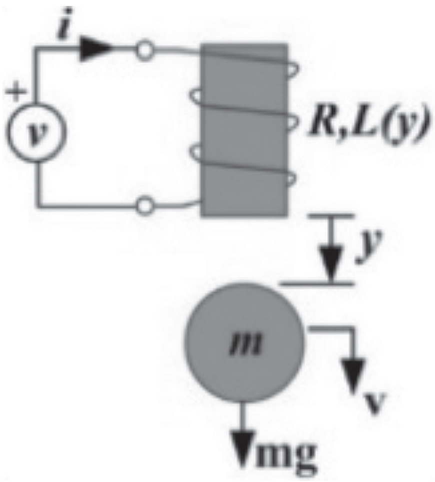
1. INTRODUCTION

With the aim of applying concepts and knowledge from control engineering, this work focuses on a Magnetic Levitation System (MLS), which consists in keeping objects suspended in the air without any mechanical contact thanks to magnetic force (figure 1). This is a nonlinear and unstable system by nature, which imposes a greater difficulty with respect to systems dealt with in other control subject areas already discussed [1][2].

Figure 1. Magnetic Levitation System (MLS).
Figura 1. Sistema Levitador Magnético (SLM).



Currently expectations about these systems are especially high in the field of engineering and the main interest in using magnetic levitation in applied engineering lies in the fact that they do not require lubrication because of their lack of contact, and that their maintenance costs are very low.

This lack of contact avoids friction and its derived problems (wear, heating). This feature makes magnetic levitation ideal to be used in magnetic bearings and as part of windmill turbines. Its best known application is probably its use in the suspension of magnetic levitation trains, as in the case of Japan, allowing for trains to reach speeds of up to 580km/h. There are other important applications, not as popular as the former, and thus our interest to familiarize ourselves with this topic.

Thus the Magnetic Levitation System, modeling and mainly control design is very difficult, because the Magnetic levitation system is an example of nonlinear, open loop unstable system with fast dynamics [3], however, Magnetic Levitation System has wide application in various industries than high-speed trains, frictionless bearing, etc and therefore this field of research is devoted significant effort in recent years.

2. MAGNETIC LEVITATING MODEL

2.1 Model development

To obtain the model two laws are used; one for the movement and one for the energy balance. These are Newton's second law and Kirchhoff's voltage law, respectively [4].

$$m\ddot{y} = -k\dot{y} + mg + F(y, i) \qquad (1)$$

Equation 1 refers to Newton's second law where, "m" is the ball's mass, "y" is the vertical position (y>0), "k" is the viscous friction coefficient, "g" is gravity acceleration and F(y, i) is the electromagnetic force. Electromagnet's inductance depends on the ball's position, and its model is presented in equation 2 as:

$$L(y) = L_1 + \frac{L_0}{1 + \frac{y}{a}} \qquad (2)$$

$$\therefore L_0;\ a;\ L_1 \text{ are constant}$$

Taking the energy stored in the coil as shown in equation 3, the electromagnetic force is represented by equation 4.

$$E(y, i) = \frac{1}{2} L(y) i^2 \qquad (3)$$

$$F(y, i) = \frac{\partial E}{\partial y} = \frac{L_0 i^2}{2a(1 + \frac{y}{a})^2} \qquad (4)$$

By Kirchhoff's voltage law (equation 5), "R" is the circuit resistance and "ϕ" is the magnetic flow (equation 6), as shown below:

$$v = \dot{\varphi} + Ri \qquad (5)$$

$$\varphi = L(y)i. \qquad (6)$$

Replacing the above equations and taking a x_1=y, x_2=y ̇ and x_3=i as state variables, and u=v as input, the system is defined by:

$$\dot{x_1} = x_2 \qquad (7)$$

$$\dot{x_2} = g - \frac{k}{m} x_2 - \frac{L_0 a {x_3}^2}{2m(a + x_1)^2} \qquad (8)$$

$$\dot{x_3} = \frac{1 + \frac{x_1}{a}}{L_1\left(1 + \frac{x_1}{a}\right) + L_0} \left[-Rx_3 + \frac{L_0 a x_2 x_3}{(a + x_1)^2} + u \right] \qquad (9)$$

2.2 Linearization

The procedure consists first in finding the equilibrium point and then in doing a linearization of the form

$\dot{x} = Ax + Bu$ [5][6].

Operating point:

An operating point is established at a position of $x_{1e}=y_e$ and taking into account the definition of equilibrium point $\left(\frac{df(x_{1e})}{dt}=0\right)$. Replacing this in equation 7, we find:

$$x_{2e} = 0 \tag{10}$$

Evaluating x_{2e} and x_{1e} in equation 8, it is found that x_{3e} is equal to:

$$x_{3e} = \sqrt{\frac{2gm}{L_0 a}}(a + x_{1e}) \tag{11}$$

Doing the analogous procedure in equation 9, that is to say, evaluating x_{1e}, x_{2e} and x_{3e} and expanding u, we arrive at:

$$u_e = Rx_{3e} \tag{12}$$

In summary, our operating point is defined by equations 13 and 14 shown below:

$$\mathbf{X_e} = \begin{bmatrix} x_{1e} \\ x_{2e} \\ x_{3e} \end{bmatrix} = \begin{bmatrix} y_e \\ 0 \\ \sqrt{\frac{2gm}{L_0 a}}(a + x_{1e}) \end{bmatrix} \tag{13}$$

$$u_e = R\sqrt{\frac{2gm}{L_0 a}}(a + x_{1e}) \tag{14}$$

- Linearization:

Below the system's parameters are presented, then an equilibrium point is chosen and, finally, linearization is done.

Table 1. System's parameters [7].
Tabla 1. Parámetros del sistema [7].

Parameter	Value	Units
m	0.01	kg
k	0.001	$\frac{N}{(m/s)}$
g	9.81	m/s^2
L_0	0.01	H
L_1	0.02	H
a	0.05	m
R	10	Ω

Linearization is proposed around an equilibrium position y_e=0.05m. Replacing the operating point in equations 13 and 14, it is defined as:

$$X_e = \begin{bmatrix} 0.05 \\ 0 \\ 1.98 \end{bmatrix} \; y \; u_e = 19.8 \tag{15}$$

Finally, linearization around the previously found equilibrium point is expressed as:

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 196.2 & -0.1 & -9.9 \\ 0 & 3.96 & -400 \end{bmatrix} \tag{16}$$

$$\boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ 40 \end{bmatrix} \tag{17}$$

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{18}$$

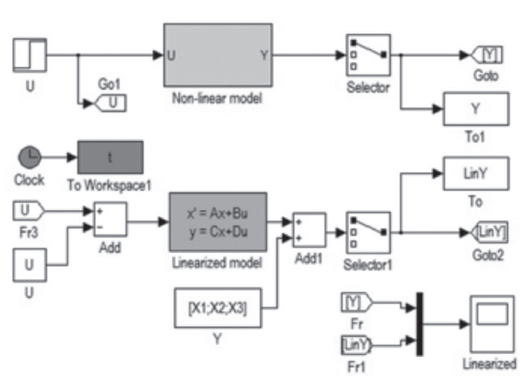
$$\boldsymbol{D} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{19}$$

From equation 16 we evaluate stability, finding A's proper values, which are S_1=13.9, S_2=-14.11 and S_3=-399.9. As it has a positive value, this means that the system has a pole on the right-hand semi-plane and thus it is unstable [8].

2.3 Comparison of linear and nonlinear models

Using Matlab-Simulink is made the comparison of models non-linear and the linearized with changes at the entrance, in order to check the response of the system with an excited initial state (figure 2) [9][10].

Figure 2. Comparison of the non-linear and linearized model.
Figura 2. Comparación del modelo no lineal y linealizado.



To verify this and the other points, the Simulink tool provided by Matlab software is used. The responses presented below are given in a 10% change (increase) of the input from its nominal value.

Figure 3. Nonlinear model vs linearized model with respect to the first variable of state.
Figura 3. Modelo no lineal vs modelo linealizado con la primera variable de estado.

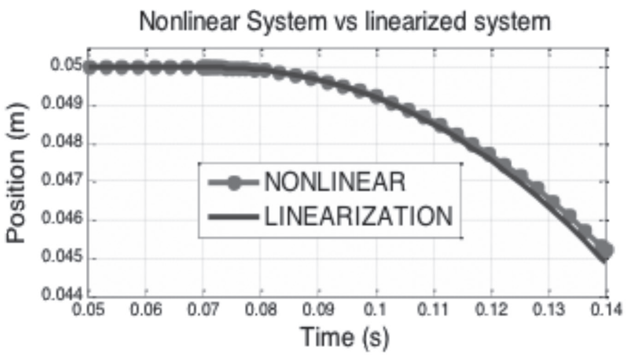


Figure 4. Nonlinear model vs linearized model with respect to the second variable of state.
Figura 4. Modelo no lineal vs modelo linealizado con la segunda variable de estado.

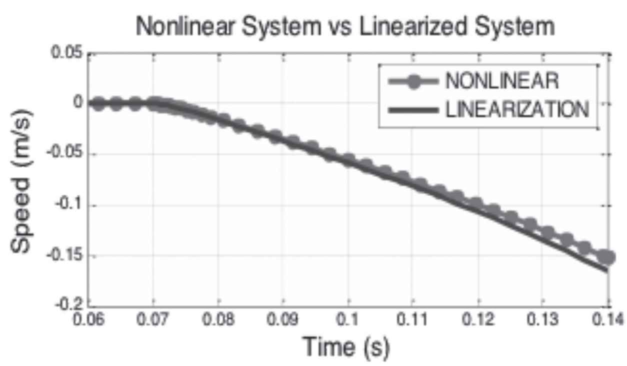
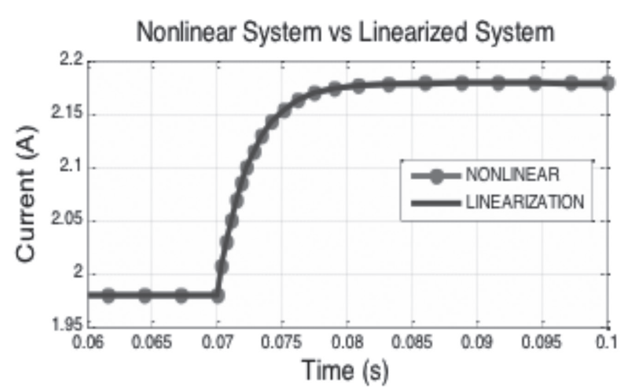


Figure 5. Nonlinear model vs linearized model with respect to the third variable of state.
Figura 5. Modelo no lineal vs modelo linealizado con la tercera variable de estado.



In conclusion, with respect to linearization, it can be observed that in the three figures shown above (figures 3, 4 and 5) the linear model follows the nonlinear one for small changes in input. For this reason, linearization is the right choice and it represents the nonlinear model correctly.

3. CONTROLLABILITY AND OBSERVABILITY

Since our system is unstable, a K for LFSV must be found such that it stabilizes the system. For this, an establishing time (Et) of 0.28 seconds and an over-percentage level (Op) of 0.02 % are proposed [11]. According to the system (MLS) the measured variables are related to the state variables; the current (i), the height and the speed (y), the controlled variable is the voltage source that powers the system.

3.1 Controllability

Controllability verification

For this, the controllability matrix and its respective row range are presented.

$$M_{AB} = \begin{bmatrix} 0 & 0 & -0.0004 \\ 0 & -0.0004 & 0.1585 \\ 0 & -0.0160 & 6.3984 \end{bmatrix} * 10^6 \quad (20)$$

$$\therefore \; Range(M_{AB}) = 3$$

Since the controllability matrix range is equal to the plant order, it can be concluded that the system is controllable.

- Desired polynomial

For this, the relationship of over-percentage level and establishing time is presented below:

$$Sp = 100e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \quad (21)$$

$$Ts = \frac{4}{\xi w_n} \quad (22)$$

From the equations above ((21) and (22)) we find the value ξ y w_n which meets the establishing time (Et) and the desired over-impulse (Oi). Thus, the polynomial will have the following structure:

$$P_1 = s^2 + 2\,\xi w_n + {w_n}^2 \quad (23)$$

Since the order of the plant is 3 and we have an order 2 polynomial (equation 23), a pole must be added which has the following structure:

$$P_2 = (s + D_s \xi w_n) \quad (24)$$

Where ξw_n is the real part of the desired polynomial's poles in equation 23, and D_s is the distance where the additional pole will be placed, such that the desired behavior is not modified. Thus, the desired polynomial will be as follows:

$$Pd = (s^2 + 2\,\xi w_n + w_n{}^2)(s + K\xi w_n) \quad (25)$$

K and Ak of LFSV

Finally we need to find K such that the proper values of the matrix A_k=(A-KB) are negative. For this, the Ackerman formula was used [12]:

$$K = [\ldots\ldots] M_{AB} P_d(A) \quad (26)$$

With the help of Matlab, equation 26 is resolved, which yields K of LFSV, and subsequently Ak is calculated, which results in:

$$K = [-75.84 \quad -5.08 \quad -7.86] \quad (27)$$

$$A_k = \begin{bmatrix} 0 & 0.001 & 0 \\ 0.1962 & -0.0001 & -0.0099 \\ 3.0336 & 0.2072 & -0.0856 \end{bmatrix} * 10^3 \quad (28)$$

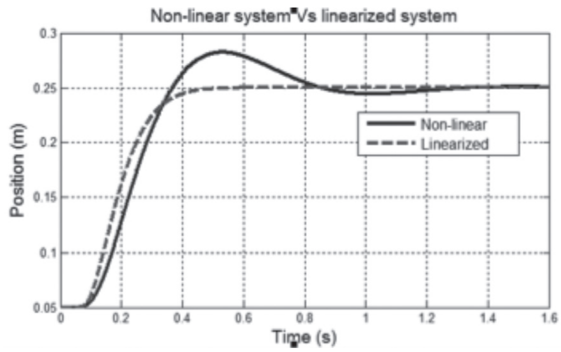
Since A_k defines the system's dynamics with LFSV, we find the proper values, which are: S_1= -57.1429, S_2= -14.2857 + 5.2693i and S_3= - 14.2857 -5.2693i. As can be observed, the proper values of A_k are lower than zero, by which it is demonstrated that linear feedback in state variables makes the system stable.

LFSV's graphic verification

The LSFV controller (Linear Feedback in State Variables) required information for each of the states of the plant or system, to determine the control action as a linear combination of the states.

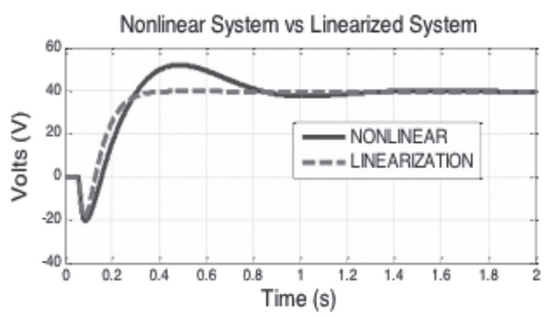
The response in LFSV for both the linear and nonlinear system is presented. According to this, it is clear that the LFSV stabilized the system. For the linear system, the proposed behavior is met, but this is not the case for the nonlinear one, in which a greater over impulse and a longer establishing time can be seen [13]. Nevertheless, the controller by LFSV is acceptable (figure 6).

Figure 6. Response LFSV for the system linear and non-linear.
Figura 6. Respuesta RLVE para el sistema lineal y no lineal.



The initial undershoot observed, refers to the control effort required to place the ball in the equilibrium position (figures 7 and 11). Control action used for LFSV, both from linear and non linear systems, is presented. As can be observed, the nonlinear system has a greater over-impulse than the linear one (figure 7).

Figure. 7. Control action for the linear system and non-linear controller using the LSFV.
Figura. 7. Acción de control para el sistema lineal y no lineal usando el controlador RLVE.



3.2 Observability

For the observer calculation, an analogous procedure to the one carried out for the calculation of the controller by LFSV is used. A dynamics faster than LFSV must be established and thus an establishing time (Et) of 0.15 seconds, an over-percentage level (Op) of 0.1 % and a distance of 8 where the additional pole will be placed, are chosen.

Observability verification

For this, the observability matrix and its column range are found:

$$M_{AC} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 192.2 & -0.1 & -9{,}9 \end{bmatrix} \quad (29)$$

$$\therefore \; Range(M_{AC}) = 3$$

Since the matrix range is equal to the system's order, then it is observable [14].

- Ke calculation

Proposing a desired polynomial as in equation 25 and resolving Akerman equation, we obtain:

$$K_e = \begin{bmatrix} -0.0002 \\ 0.0701 \\ 2.8159 \end{bmatrix} x10^6 \tag{30}$$

Finally, the observer's form is defined as:

$$\dot{\hat{x}} = A\hat{x} + Bu + K_e(y - \hat{y}) \tag{31}$$

$$\hat{y} = C\hat{x} \tag{32}$$

- Observer verification

To verify the observer two methods are used: an analytical and a graphic one. For the former, Ake is proposed, in which it must be checked that the error is asymptotically stable.

$$A_{ke} = (A - K_eC) \tag{33}$$

$$A_{ke} = \begin{bmatrix} 0.0002 & 0 & 0 \\ -0.0699 & 0 & 0 \\ -2.8159 & 0 & -0.0004 \end{bmatrix} x10^6 \tag{34}$$

For the error to be asymptotically stable, Ake must be Hurwis (all the proper values with negative real part).

$$S = \begin{bmatrix} -2.1333 \\ -0.1333 + 0.0455\mathrm{i} \\ -0.1333 - 0.0455\mathrm{i} \end{bmatrix} x10^2 \tag{35}$$

As shown in the above equation, all Ake's proper values have negative real part, which indicates that the error is asymptotically stable and our observer is suitable.

Next we continue with graphic verification, which is performed in open-loop and for LFSV.

The comparison of linear model output with open-loop observer. As can be observed, one is above the other, so it is a good state estimator (figure 8). The comparison of nonlinear model output with open-loop observer. They are very similar and only at the end it can be seen how they separate; but for small changes it is a good state estimator (figure 9).

Figure 8. Comparison of output of the linear model with observer for open-loop.
Figura 8. Comparación de salida del modelo lineal con observador de lazo abierto.

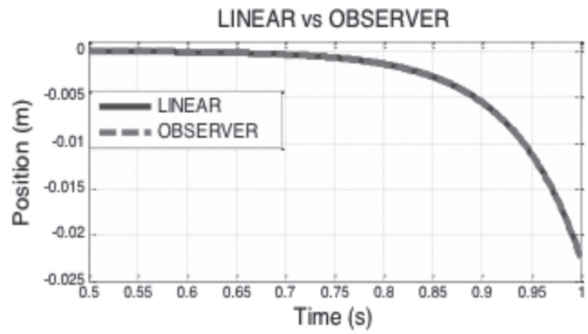
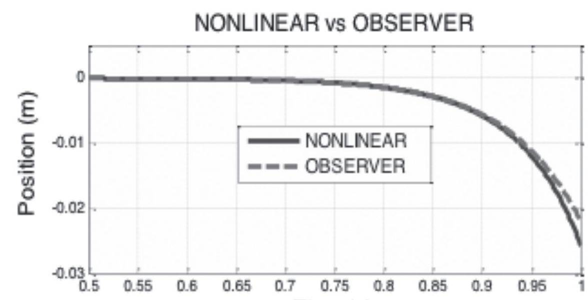
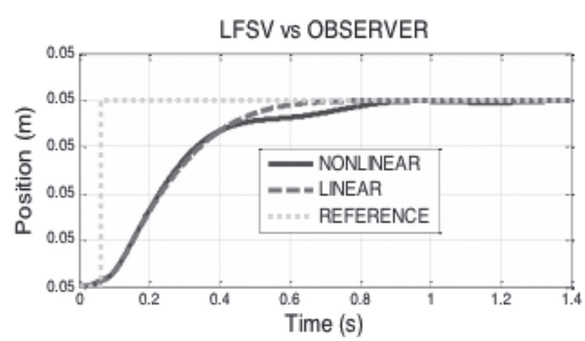


Figure 9. Comparison of the output of the nonlinear model with the observer in open loop.
Figura 9. Comparación de la salida del modelo no lineal con el observador en lazo abierto.



It compares the observer with LFSV for both linear and nonlinear systems. For the former, it can be observed that the response is similar to the LFSV without observer. For the nonlinear, greater error and establishing time can be seen. Nevertheless, the observer is considered to be suitable for the magnetic levitator (figure 10).

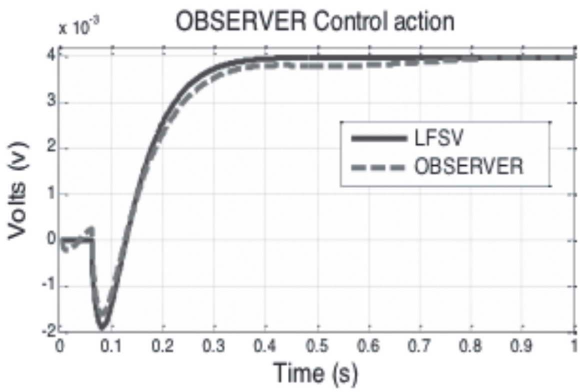
Figure 10. Comparison of the observer with RLVE system, with the linear and non linear.
Figura 10. Comparación del observador con RLVE, con el sistema lineal y no lineal.



Control action of LFSV, with and without observer, in which it can be seen that both signals are very similar.

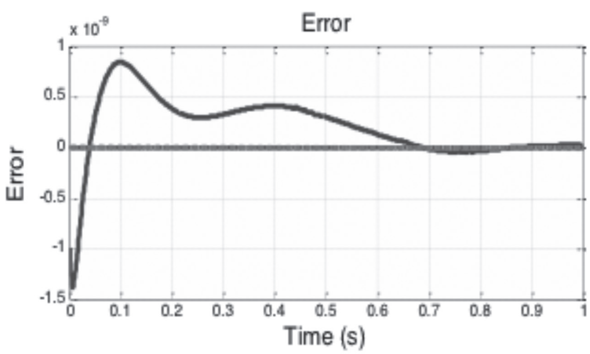
The only difference is that the observer's control effort takes longer to reach the stationary state (or nominal value), (figure 11).

Figure 11. Control action with RLVE observer without observer.
Figura 11. Acción de control, de la RLVE con observador y sin observador.



It represents the observer's estimation error. As can be seen, it is greater at the beginning because the observer's initial conditions are different from zero, but at the end this error approaches zero and thus theory and simulation coincide; the error is zero asymptotically (figure 12).

Figure 12. Error of estimation of the observer.
Figure 12. Error de estimación del observador.



3.3 Optimal Linear Quadratic Regulator (LQR)

The LQR controller (linear quadratic regulator) controller seeks to minimize the energy present in the system, and the control that is obtained by minimizing this criterion is linear.

The method consists in finding a K for feedback in state variables such that it minimizes the following functional cost [15]:

$$J = \int_0^{\infty} [X^T QX + U^T RU)dt \qquad (36)$$

For this, the following Q and R are proposed:

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } R = 1 \qquad (37)$$

To find an optimal K, Riccati equation must be resolved (equation 38).

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \qquad (38)$$

Finally, the optimal K will be equal to:

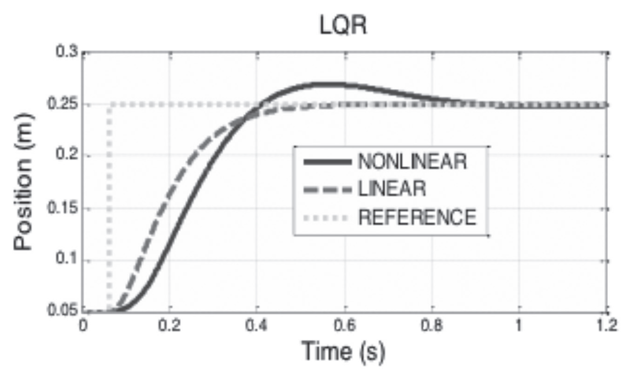
$$Koptimal = R^{-1}B^T P \qquad (39)$$

Using Matlab optimal K is equal to:

$$k_{OPTIMAL} = [-409.9697 \quad -29.0826 \quad 0.6959] \qquad (40)$$

Finally, let us analyse optimal Ak's proper values, which are: S_1= -13.5, S_2= -14.5 and S_3= -399.9; as they all are on the left-hand semi-plane, the system is stable for this optimal K.

Figure 13. Comparison of the LQR controller with non-linear and linear system.
Figura 13. Comparación del controlador LQR con sistema lineal y no lineal.

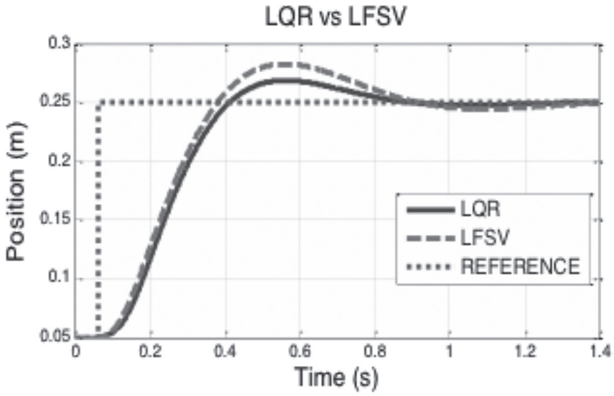


4. RESULTS AND DISCUSSION

The results are presented through the comparison of the response of the linear system and non-linear, both for the driver as to the LQR LSFV, which allows you to analyze the behavior of the system obtained through the influence of each controller.

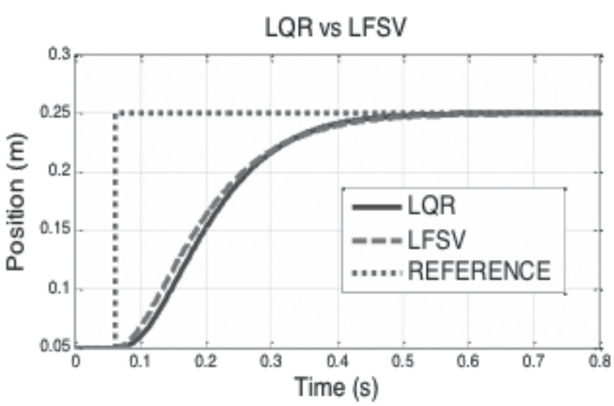
Figure 14 compares the behavior of the system controlled by the LFSV and the LQR, both applied to the non-linear model. I could see was noted that the controlled system with the LQR presents a minor on impulse that RLVE. In this outcome the LQR controller ensures a positioning of stabilization of the more soft magnetic levitator and with the same time that the establishment LFSV.

Figure 14. Comparison of the non-linear system controlled by the LFSV and the LQR.
Figura 14. Comparación del sistema no lineal controlado por el LFSV y el LQR.



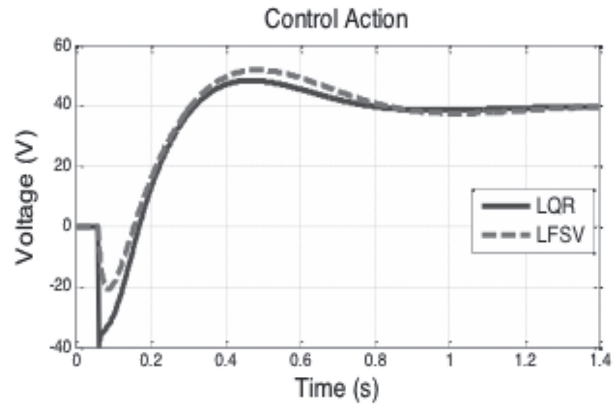
Unlike the previous result presented in figure 14, where there is a outcome in favor for the controlled system with the LQR. In figure 15 the linear system is not a significant difference between the behavior of the system compared by the drivers and LFSV LQR, it should be noted that the non-linear system is more closer to the actual behavior of the system.

Figure 15. Comparison of the linear system controlled by the LFSV and the LQR.
Figura 15. Comparación del sistema lineal controlado por el LFSV y el LQR.



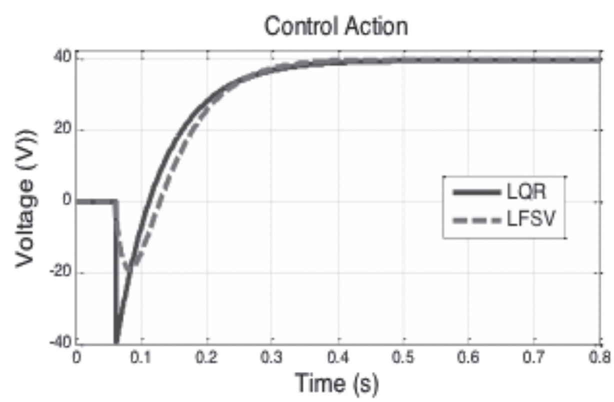
In figure 16 you can see a sub-impulse, before spending the 0.2 seconds, in which the sub-impulse of greater magnitude and presents the system controlled by LQR, this is reflected in a higher cost of energy, that's costing you more to the controller, but is subsequently offset with the on momentum between the (0.4 to 0.6) seconds less pronounced in front of the driver LFSV.

Figure 16. Comparison of the control action of the non-linear system controlled by the LFSV and the LQR.
Figura 16. Comparación de la acción de control del sistema no lineal controlado por el LFSV y el LQR.



The result obtained in figure 17, presents a similar behavior in the sub-impulse as explained in the figure 16, although it does not feature an on momentum as evidenced in the above figure, such behavior exposes that the sub-impulse is present in the linear model and non-linear without the driver, therefore it is a behavior of the system itself.

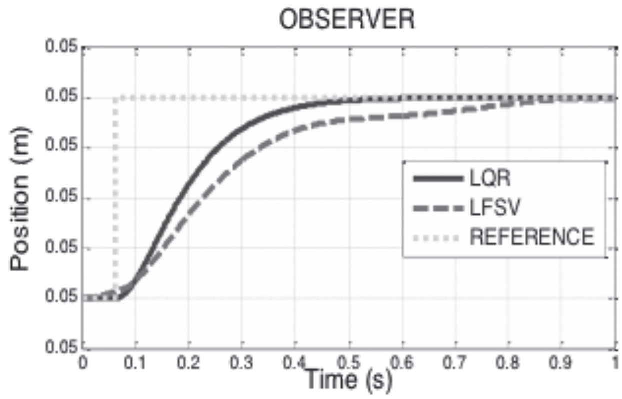
Figure 17. Comparison of the control action of the linear system controlled by the LFSV and the LQR.
Figura 17. Comparación de la acción de control del sistema lineal controlado por el LFSV y el LQR.



- LQR with observer:

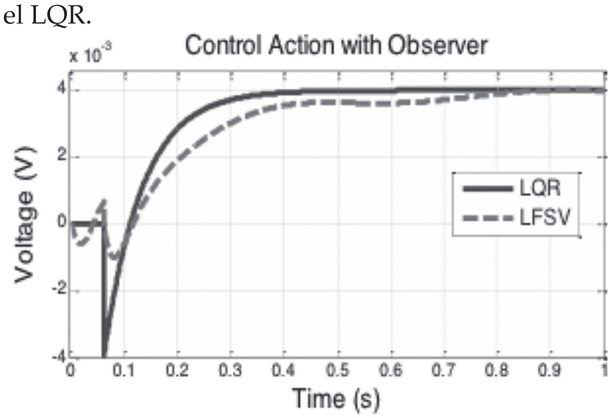
In this section, the answer with the LQR by applying the previously calculated observer is analyzed [16]. In the figure 18, a better answer is obtained with the controller LQR, which presents an establishment time between (0,4 to 0,5) seconds, while the system controlled by LFSV presents an establishment time in 0,8 seconds, which is late too much to stabilize the system. This way the behavior of the system controlled with the LQR and observer of the state presents a better performance.

Figure 18. Comparison of the non-linear system with observer, controlled by the LFSV and the LQR.
Figura 18. Comparación del sistema no lineal con observador, controlado por el LFSV y el LQR.



With respect to the control action that is presented in figure 19, the non-linear system with observer responds faster before the sub-impulse, with a time less than 0.1 seconds, and with an energy expenditure 10 times less than the submitted to the same system without observer. In this way, the system with observer presents a better behavior before the sub-impulse.

Figure 19. Comparison of the control action of the non-linear system with observer, controlled by the LFSV and the LQR.
Figura 19. Comparación de la acción de control del sistema no lineal con observador, controlado por el LFSV y el LQR.



In this way, the results for the non-linear system, when you enter a measurement noise variance of 1x10-15, where it is possible to verify the behavior of the system with regard to its position. In figure 20, this noise is negligible for the LQR controller, but you can observe a large variability in the response of the driver RLVE, which dramatically affects its stability.

In addition, it may become evident in the figure 21, the variability, the effort to control that you must have the driver LFSV to keep the system stable, something that is transformed to a large amount of energy used to control the system.

Figure 20. Response of the non-linear system with observer in a noise measurement.
Figura 20. Respuesta del sistema no lineal con observador ante un ruido de medición.

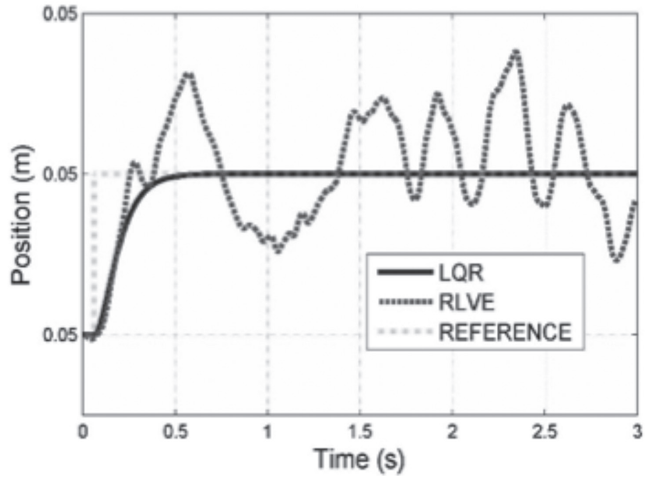
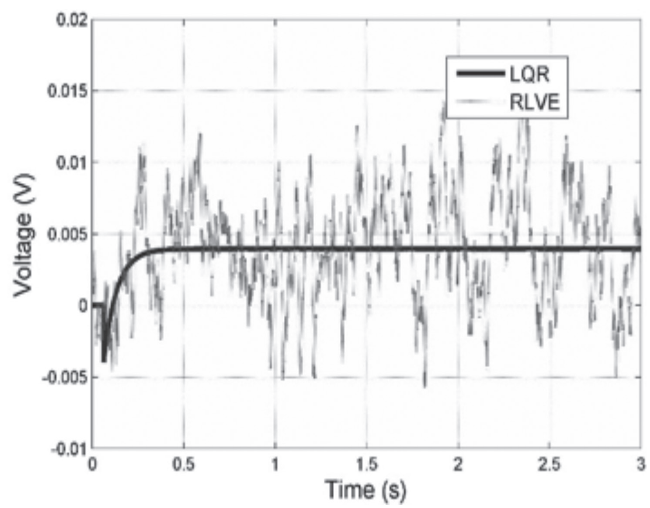


Figure 21. Control action of the non-linear system with observer in a noise measurement.
Figura 21. Acción de control del sistema no lineal con observador ante un ruido de medición.



Finally, in the table 2 is a summary of the analysis of results, in which engage system parameters such as sub-impulse, time of establishment, over-percentage, Noise measurement and control action. With the aim to highlight the behavior of the system that characterizes the system magnetic levitator and also provide important tools, which are taken as starting points for the design, modeling, control and implementation of the non-linear system of magnetic levitator. To characterize the table 2, identifies three points of measurement: Insufficient, acceptable and excellent (I, A and E), according to the behavior of the drivers submitted with respect to the parameters in this table.

Table 2. Analysis of results
Tabla 2. Análisis de resultados

Parameter	LQR	LFSV	Observation
Sub-impulse	E	A	The non-linear system controlled by the LQR presents a minor on impulse that RLVE (figure 13), plus this improved for the two controllers, adding the observer to the system (figure 19).
Establishing time	E	I	The time of establishment of the system became similar in the two controllers, but with the observer added to the system the LQR control showed a better result compared to LFSV (figure 18).
Over-percentage	A	A	On the percentage level, was presented in a similar manner for the two controllers, with a slight difference in favor of the LQR controller (figure 13).
Noise measurement	E	I	The noise of measurement became chaotic LFSV in the control, and null for the LQR controller, therefore the LQR control presents the quality of noise immunity (figure 20).
Control action,	A	E	The control action showed two important results, the first in figure 19, the LQR control presents greater consumption of energy to stabilize the system.
Control action, Noise measurement	E	I	The control action showed two important results, in second place when introduces noise measurement of the LQR control shows better performance than the LFSV (figure 21).

4. CONCLUSIONS

• Some difficulties arose in the simulation because deviated variables were not easily identified from non deviated variables.

• There were difficulties with the state observer because initial conditions must be very close to zero; otherwise, the observer's error with respect to the original will be so big as to make the system unstable, and by the time the error is zero it would be too late for the controller to do anything. A clear example would be: if the ball is levitating and the controller reduces the intensity of the magnetic force, due to an error in the observer until a point when gravity force is higher, the ball will fall to the ground, and by the time the observer's error is zero, there is no way the levitator can lift the ball from the ground.

• When analyzing the differences between the controller by LFSV and by LQR, not much difference is perceived between them. Only when measurement noise is added, the LQR tolerates noise with greater variability than the LFSV. The latter becomes unstable from a certain value, while the LQR does not.

The theory about control in state variables applied in this magnetic levitator is very interesting since an unstable system can easily become stable and different controllers can be used.

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