Mean variance portfolio selection problem with multiscale stochastic volatility

Problema de selección de cartera de la varianza media con volatilidad estocástica multiescala

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ABSTRACT

This paper discussed the mean-variance portfolio selection problem with multiscale stochastic volatility. We considered two type of volatility, including a fast –moving one and a slowly-moving one by using the stochastic dynamic programming principle and Hamilton-Jacobi-Bellman equation approach, the optimal investment strategy, the value function and the efficient frontier are derived in closed form.

Keywords: Mean-variance portfolio selection; Multiscale Stochastic Volatility; Efficient frontier; stochastic control, Probability.

RESUMEN

En este artículo se discute el problema de selección de cartera de media-varianza con volatilidad estocástica multiescala. Se considera dos tipos de volatilidad, incluida una de movimiento rápido y otra de movimiento lento. Mediante el uso del principio de programación dinámica estocástica y el enfoque de la ecuación de Hamilton-Jacobi-Bellman, la estrategia de inversión óptima, la función de valor y la frontera eficiente se derivan en forma cerrada.

Palabras Claves: Selección de cartera de media-varianza; volatilidad estocástica multiescala; Frontera eficiente; Control estocástico, Probabilidad.

INTRODUCTION

The presence of volatility factors is well documented in the literature using underlying returns data (see [1,2,3,4,5,6,7,8], for instance). While some single-factor diffusion stochastic volatility models such as [9] enjoy wide success, numerous empirical studies of real data have shown that the two-factor stochastic volatility models can produce the observed kurtosis, fat-tailed return distributions and long memory effect. For example, [1] used ranged-based estimation to indicate the existence of two volatility factors including one highly persistent factor and one quickly mean-reverting factor. [3] used the efficient method of moments (EMM) to calibrate multiple stochastic volatility factors and jump components. They showed 2 that two factors are necessary for log-linear models. [10] disscuesd a continuous –time Mean Variance portfolio selection problem with Vasicek stochastic interest rates.

Stochastic volatility models relax the constant volatility assumption of the Black-Scholes model by allowing volatility to follow a random process. In this context, the market is incomplete because the volatility is not traded and the volatility risk cannot be fully hedged using the basic instruments (stocks and bonds). To preclude arbitrage, the market selects a unique risk neutral derivative pricing measure, from a family of possible measures. As a result, in contrast to the Black-Scholes model, the stochastic volatility models are able to capture some of the well-known features of the implied volatility surface, such as the volatility smile and skew.In this paper we will consider a mean –variance portfolio selection with Multiscale stochastic volatility model.

MODEL FORMULATION

Given a filtered probability space (Ω,P,f,{f_t }_(t≥0)=σ{W(s);0≤s≤t}) be augmented by all the *P-null sets in f*, where *f = ft*. Let *W(t)* be a one dimensional standard Brownian motion define on (Ω,P,*f*) our t ≥ 0 The investor joins a market at time 0 with initial wealth $x_0 > 0$ and plans to invest dynamically over a fixed time horizon *T*.

The financial market consists of one risk-free asset and one risky asset. The price of the risk-free asset is governed by the following differential equation

$$dB(t) = B(t)r(t)dt, \quad B(0)=b_0>0$$

b_0 is the initial price of the risk-free assets, d represents the differential operators and short rate $r(t)$ satisfies the differential equation

$$dr(t) = (\overline{a}(t) + b\psi(t))dt + bdw(t), \quad r(0)=r_0.$$

The risk premium $\psi(t)$ is deterministic and continuous function, $w(t)$ is the standard Brownian motion and $\overline{a}(t) = \theta(t) - \alpha r(t)$ is a stochastic process related to $r(t)$ were $a > 0$,and θ is a deterministic and continuously differentiable function [11].

The other asset is a (zero) bond whose price process is modeled as

$$\begin{cases} dS(t) = S(t)\left[\left(r(t) + \psi(t)\mu(y(t),z(t))\right)dt + \beta(y(t),z(t))dW^0(t)\right], \\ dy(t) = \frac{1}{\xi}b(y(t)dt + \frac{1}{\sqrt{\xi}}a(y(t))dW^1(t), \\ dz(t) = \delta c(z(t))dt + \sqrt{\delta}\,g(z(t))dW^2(t), \\ S(0) = s_0 = 0, y(0) = y_0, z(0) = z_0, \end{cases}$$

were two timescale factors is applied in our stochastic volatility, which means we use $\sigma(yt\, z_t)$ instead of σ(t). s_0, y_0 and z_0 are respectively the initial price and initial volatilities of the risky asset. $\mu(y(t),z(t))$and $\beta(y(t),z(t))$ are respectively the appreciation rate and volatility rate of the risky assets price. As described in the research work by [12], the dynamics of y(t) and z(t) respectively shows the fast and slow variation of volatility with very small values of parameters ξ, and δ. Besides, we assume that the process $y(t) = y'(1/\xi)$ in distribution where y'(t) is an ergodic diffusion process with unique invariant distribution $\overline{\phi}$, independent of ξ . As described in [13], we denote ⟨.⟩ as the invariant expectation with respect to $\overline{\phi}$

$$\langle g \rangle = \int g(y)\phi\,(dy).$$

The standard Brownian motion, $w^0(t)$, $w^1(t)$ and $w^2(t)$ are correlated with

$Cov\,(w^0(t),w^1(t)) = \rho 1$, $Cov(w^0, w^2(t)) = \rho_2$, $Cov\,(w^1(t),w^2(t)) = \rho_3$ where $-1<\rho_1<1$, $-1< \rho_2<1$, $-1<\rho_{12}<1$ and $1+ 2\rho_1\rho_2\rho_{12} - \rho_1^2 - \rho_2^2 - \rho_{12}^2 > 0$, to ensure positive definiteness of the covariance matrix of the three Brownian motions. (15)

Here the appreciation rate is $(r(t) + \psi(t))\,\sigma(t) = \mu$ which results from the fact that the risk premium $\psi(t)$ is defined as

$$\frac{\mu(t) - r(t)}{\sigma(t)}$$

Let $\pi(t)$ denote the amount invested in the (zero) bond at time t, ($t \in [0,T]$), and $\pi(t)$ the wealth at time t corresponding to investment strategy π. Then the wealth process satisfies the following stochastic differential equation:

$$dX^{\pi} = \pi(t)\,((r(t) + \psi(t)\alpha\,(y(t), z(t))dt + \beta(y(t)), z(t)dw^{0}(t)\,] + (\,X^{\pi}(t) - \pi(t))r(t)\,dt$$

$$=[\pi(t)\psi(t)\alpha\big(y(t), z(t)\big) + X^{\pi}(t)r(t)]dt + \pi(t)\beta\big(y(t), z(t)\big)dw(t) \qquad (1)$$

with initial condition $X(0)= x_{0.}$

The following Mean-variance portfolio selection problem will be discussed in this paper

$$P(w) = \begin{cases} \min\limits_{\pi(.)\epsilon\Pi(0,x_0)} E[X^{\pi}(T) - w]^2, \\ s.e\; E(X^{\pi}(T)) = w \end{cases}$$

where w is a premium return level and $\pi(0, x_0)$ denotes the set of all admissible controls defined as in [14]

By convex optimization theory, problem $P(w)$ can be solved via the following optimal stochastic control problem with a Lagrange multiplier 2λ

$$PL1(\lambda, w) \min_{\pi(.)\epsilon\Pi(0,x_0)} \{E[X^{\pi}(T) - w]^2 - 2\lambda[E(X^{\pi}(T) - w)]\}$$

The relationship between the optimal solutions of these two problems is concluded in the following lemma (see [14]).

Lemma 1 Denote by $\Gamma(\lambda)$ and $\pi\hat{}(\lambda, t, x(t), y(t), z(t), t\epsilon[0,T])$, respectively, the optimal value and the optimal strategy of problem $1(\lambda, w)$. Then the optimal value and the optimal strategy of problem $P(w)$ are

$$\sup_{\lambda\epsilon R}\Gamma(\lambda)\;\; and\;\; \{\hat{\pi}(\lambda, t, x(t), y(t), z(t), t\epsilon[0,T])\}$$

respectively, where $\lambda^{*} = arg \sup\limits_{\lambda\epsilon R}\Gamma(\lambda)$

The objective function of problem $PL1(\lambda, w)$ can be rewritten as $E[X^{\pi}(T) - (\lambda + w)]^2 - \lambda^2$ the solution of problem $PL1(,w)$ is equivalent to that of the problem

$$PL2\,(\lambda, w) \min_{\pi(.)\epsilon\Pi(0,x_0)} E[X^{\pi}(T) - w]^2$$

For problem $PL2$ we define the value function.

$$V(t, x, y, z, r) = \min_{\pi(.)\epsilon\Pi(t,x,y,z)} E[(X^{\pi}(T) - (\lambda + w)^2 | X(t) = x, z(t) = z, y(t) = y]$$

Using Hamilton-Jacobi-Bellman's Optimality principle

$$V(t, x, y, r) = \min_{\pi(.)\epsilon(t,x,)} E[V(t+h), X^{\pi}(t+h), r(t+h)]\;\forall h > 0$$

Define an operator

$$\begin{aligned} A^{\pi}V(t, x, y, z, r) &= V_t + V_x\,[\pi(t)\psi(t)\mu(y,z) + X(t)r(t)] + \frac{b(y)}{\xi}V_y + \delta\,C(z)V_z \\ &+ \frac{1}{2}V_{xx}\pi^2\beta^2(y,z) + \frac{1}{2\xi}V_{yy}\mu^2(y) + \frac{1}{2}g^2(z)\delta V_{zz} \\ &+ \frac{\pi(t)\beta(y,z)\mu(y)P_1}{\sqrt{\xi}}V_{xy} + \pi(t)\beta(y,z)g(z)P_2\sqrt{\sigma V_Z} \\ &+ \sqrt{\frac{\delta}{\xi}}\mu(y)g(z)P_{1,2}V_{yz} + \frac{b}{\sqrt{\xi}}a\big(y(t)\big)V_{yr} + b\sqrt{\delta}g\big(z(t)\big)V_{zr} \\ &+ b\pi(t)\beta(y,z)V_{rx} + \frac{1}{2}V_{rr}b^2 + V_r(\bar{a}(t) + b\psi(t)). \end{aligned}$$

Using the Ito formular the following equation is obtained.

$$V(t+h, X^{\pi}(t+h), z(t+h), y(t+h), r(t+h)) = V(t,x,y,z,h) + \int_t^{t+h} A^{\pi(s)} V(s, X(S) y(s), z(s)\ r(s)) ds + \int_t^{t+h} (bV_r + V_x \pi(s) \beta(y(s), z(s)) dw(s) \quad (eg3)$$

According to presented by [11], if $E_{(t,x)}$ $[(\int_t^{t+h}$ $(bV_r + V_x\ \pi\ (s)\beta\ (y(s),\ z(s))\ )^2\ ds] <+\infty$, then $(\int_t^{t+h}$ $(bV_r + V_x\ \pi\ (s)$ $\beta\ (y(s),\ z(s))dw(s)\ )$ is a martigale.therefore when $E_{t,x}$ $[\int(bV_r+V_x\ \pi\ (s)\ \beta(y(s)z(s))\ ] < +\infty$ substituting (3) into (2) yields the HJB equation of *V(t,x,y,z,r)* as follows.

$$V_t + V_x xr + \frac{b(y)}{\xi} V_y + \delta C(z) V_z + \frac{1}{2\xi} a^2(y) V_{yy} + \frac{1}{2} g^2 \delta V_{zz} + \sqrt{\frac{\delta}{\xi}} a(y) g(z) p_{12} V_{yz} + \frac{b}{\sqrt{\xi}} a(y(t)) V_{yr} + b\sqrt{\delta} g(z) V_{zr} + \frac{1}{2} b^2 V_{rr} + (\bar{a}(t) + b\psi(t)) V_r + \min_{\pi(t)} \left\{ \frac{1}{2} \pi^2 \beta^2(y,z) V_{xx} + \frac{\pi \beta(y,z) a(y) p_1}{\sqrt{\xi}} V_{xy} + \pi(t) \beta(y,z) g(z) p_2 \sqrt{\delta} V_{xz} + b\pi(t) \beta(y,z) V_{rx} + \pi(t) \psi(t) \beta(y,z) V_x \right\} = 0 \qquad (4)$$

With the boundary condition $(V,x,y,z,r) = (xyz - (\lambda + w))^2$.

We derive the following theory in order to know the contribution of the HJB equation (4) to derive the optimal strategy and the value function.

Theorem 1 suppose that $v(x,y,z,r)$ $\epsilon C^{1,2,3}$ ([0,T]X 0)where $0 \sqsubseteq R^2$,satisfies *(i)v (t,x,y,z r) solves* (4) with boundary condition;

(ii) for any admissible control π(.) and its corresponding wealth process,

$$E_{t,x}\left[\left(\int_t^{t+h} (bV_r + V_x \pi(s) \beta(y(s), z(s))\right)^2 ds\right] < +\infty, \forall t \epsilon [0,], \mathrm{h} > 0$$

iii) For all sequences of stopping times $(\tau_n : 0 \le \tau \le T)_{n \in N}$ and any admissible strategy π(.) ϵ∏ (0,x_0), the sequence $\{v(\tau_n, X^{\pi}(\tau_n))\}_{n \epsilon N}$ is uniformly integrable.

Then we have

a) $v(t,x,y,z.r) \le V(t,x,y,z,r)$;

b) if there exists an admissible strategy $\hat{\pi}(t) \in argmin\ A^{\pi}\ v(t, X^{\pi}(t), r)$, then $v(t,x,y,z,r) = V(t,x,y,z,r)$.

We are going to work to deduce the optimal strategy and the value function of problem *PL*2(λ,w).

OPTIMAL SOLUTION

Assume that *v* is a solution of HJB (4) when $Vxx > 0, Vzz, Vyy > 0$ the optimal strategy of problem *PL*2(λ,*w*) is

$$\hat{\pi} = -\frac{\left(a(y) P_1 V_{xy} + g(z) p_2 \sqrt{\delta} V_{xz} + bV_{rx} + \psi(t) V_x\right)^2}{\beta(y,z) V_{xx}} \qquad (5)$$

Inserting equation (5) back to equation (4) yieds

$$V_t + V_x x r + \frac{b(y)}{\xi} V_y + \delta C(z) V_z + \frac{1}{2\xi} a^2(y) V_{yy} + \frac{1}{2} g^2(z) \delta V_{zz} + \frac{\sqrt{\delta}}{\xi} a(y) g(z) P_{12} V_{yz}$$
$$+ \frac{b}{\sqrt{\xi}} a(y(t)) V_{yr} + b\sqrt{\delta} g(z(t)) V_{Zr} + \frac{1}{2} b^2 V_{rr} + V_r(\bar{a}(t) + b\psi(t)) \quad (6)$$
$$-\frac{1}{2} \frac{\left(\frac{a(y)P_1}{\sqrt{\xi}} V_{xy} + g(z) P_2 \sqrt{\delta} V_{xz} + b V_{rx} + \psi(t) V_x\right)^2}{\beta(y,z) V_{xx}} = 0$$

With terminal condition $v(T,x,y,z,r) = (xyz - (\lambda + w))^2$.

We can verify that v has the form $v(t,x,y,z,r) = P(t,r)x^2y^2z^2 - 2(\lambda + w)Q(t,r)xyz + (\lambda + w)^2 R(t,r)$

With $P(t,r) > 0, P(T,r) = 1, Q(T,r) = 1, R(T,r) = 1$.substituting the above expression into equation (6) we obtain the following partial differential equation for $P(t,r)$, $Q(t,r)$ and $R(t,r)$ respectively:

$$\begin{cases} P_t + 2rP + \frac{b(y)P}{\xi} + \delta C(z) P + \frac{1}{2\xi} a^2(y) P + \frac{1}{2} g^2 \delta P + \sqrt{\frac{\delta}{\xi}} a(y) g(z) P_{12} P + \frac{b}{\sqrt{\xi}} a(y(t)) P \\ + b\sqrt{\delta} g(z(t)) P_r + \frac{1}{2} b^2 P_{rr} + (\bar{a} + b\psi(t)) P_r - \frac{\left[a(y)P_1 P(t,r) + g(z) P_2 \sqrt{\delta} P(t,r) + bP_r + \psi(t) P\right]^2}{P} = 0 \\ P(t,r) > 0, P(T,r) = 1 \end{cases} \quad (7)$$

$$\begin{cases} Q_t + rQ + \frac{b(y)Q}{\xi} + \delta C(z) Q + \frac{1}{2\xi} a^2(y) Q + \frac{1}{2} g^2 \delta Q + \sqrt{\frac{\delta}{\xi}} a(y) g(z) Q_{12} Q + \frac{b}{\sqrt{\xi}} a(y(t)) Q_r \\ + b\sqrt{\delta} g(z(t)) Q_r + \frac{1}{2} b^2 Q_{rr} + (\bar{a} + b\psi(t)) Q_r - \frac{\left(\psi(t)P + bP_r + g(z)P_2\sqrt{\delta}P + \frac{a(y)P_1 P}{\xi}\right)\left(\psi(t)Q + bQ_r + g(z)Q_2\sqrt{\delta} + \frac{a(y)P_1 Q}{\xi}\right)}{P} = 0 \\ Q(t,r) = 0, Q(T,r) = 1 \end{cases} \quad (8)$$

$$\begin{cases} R_t + \frac{b(y)}{\xi} a(y(t)) + \frac{b}{\sqrt{\xi}} a(y(t)) Q_r + b\sqrt{\delta} g(z(t)) R_r + (\bar{a} + b\psi(t)) R_r \\ + \frac{1}{2} b^2 R_{rr} - \frac{\left(a(y)P_1 Q + g(z)\sqrt{\delta} Q + bQ_r + \psi(t) Q\right)}{P} = 0 \\ R(T,r) = 1 \end{cases} \quad (9)$$

Let $P(t,r)$and $Q(t,r)$ be of the form

$P(t,r) = e^{A(t)r + B(t)}$ (10)

$Q(t,r) = e^{C(t) + D(t)}$ (11)

With terminal conditions

$A(T) = B(T) = C(T) = D(T) = 0$ (12)

Deducing the expression of $A(t)$, $B(t)$, $C(t)$ and $D(t)$ from the above equation inserting (10) into (7) gives

$$(A'(t) - \alpha A(t) + 2) r(t) + \beta'(t) - \frac{1}{2} b^2 A^2 + \left(\theta + b\sqrt{\delta} g(z(t)) + b\psi(t)\right) A - \left(\frac{a(y)}{\xi} P_1\right)^2 - \left(g(Z) P_2 \sqrt{\delta}\right)^2 - \psi^2 = 0$$

Substituting (11) into (8) results in

$$[C(t) - \alpha(t) + 1] r(t) + 2D' + 2\delta C(z) + \frac{a^2(y)}{\xi} + g^2(z)\delta + 2\sqrt{\frac{\delta}{\xi}} g(z) a(y) P_{12}$$
$$+ 2\theta(t) C(t) + 2b\sqrt{\delta} g(z(t)) C(t) + 2bg(z) P_2 \sqrt{\delta} C(t) + b^2 C^2(t)$$
$$- 2b^2 C(t) A(t) - 2bg(t)\sqrt{\delta} P_2 A(t) - 2b \frac{a(y)}{\sqrt{\xi}} P_1 A(t) - [2bA(t) + \psi] + 2(g(z)\sqrt{\delta} P_2)^2 + 2\left(\frac{a(y)}{\sqrt{\xi}} P_1\right)^2$$

Bringing this two equations together with terminal condition (12) results in

$$A(t) = \frac{2}{\alpha}(1 - e^{\alpha(T-t)}) = 2C(t)$$

$$B(t) = \int_t^T \left[\frac{1}{2} b^2 A^2 + (\theta(s) - b\sqrt{\delta} g(z(s) + b\psi(s)))A - \left(\frac{a(y)}{\xi} P_1\right)^2 - \left(g(z)P_2\sqrt{\delta}\right)^2 - \psi^s(s) \right] \quad (14)$$

$$2D(t) = \int_t^T 2\theta(t)C(s) + 2b\sqrt{\delta} g(z(t))C(t) + 2bg(z)P_2\sqrt{\delta}C(t) + b^2C^2(t) - 2b^2C(t)A(t) -$$
$$2bg(z)\sqrt{\delta}P_2A(s) - 2b\frac{a(y)}{\sqrt{\xi}}P_1A(t) - [2b - A(t) + \psi]\psi - 2\left(g(z)\sqrt{\delta}P_2\right)^2 - 2\left(\frac{a(y)}{\sqrt{\xi}}P_1\right)^2 \quad (15)$$

$$2D(t) - B(t) = -\int_t^T (\psi(s) + bC(s))^2\, ds + g(z)\sqrt{\delta}P_2 + \left(\frac{a(y)}{\sqrt{\xi}}P_1\right)^2 \quad (16)$$

By using the expressions of $P(t,r)$ and $Q(t,r)$ equation 9 becomes

$$R_t(t,r) + \frac{b}{\sqrt{\xi}}a(y) + b\sqrt{\delta}g(z) + (\bar{a} + b\psi(t))R_r(t,r) + \frac{1}{2}b^2 R_{rr}(t,r) - \left(\psi(t) + bC(t) + \frac{a(y)}{\sqrt{\xi}}P_1 + g(z)\sqrt{\delta}P_2\right)^2 e^{2D(t)-B(t)} = 0 \quad (17)$$

So the solution of the above equation can be of the

$$R(t) = e^{2D(t)-B(t)} = e^{-\int_t^T\left(\psi(s)+bC(s)+\frac{a(y)}{\sqrt{\xi}}+g(z)\sqrt{\delta}\right)^2 ds} \quad (18)$$

The candidate for the optimal strategy has the form

$$\hat{\pi} = \frac{(\lambda + w)\left(\psi(t) + bC(t) + \frac{a(y)}{\sqrt{\xi}}P_1 + g(z)\sqrt{\delta}\right)}{\delta} e^{-C(t)+D(t)} - \frac{\left(\psi(t) + bA(t) + \frac{a(y)}{\sqrt{\xi}}PP_1 + g(z)\sqrt{\delta}\right)}{\sigma(t)} x(t) \quad (19)$$

And the respective candidate for the value function is of the form

$$V(t,x,y,z,r) = e^{A(t)r+B(t)}x^2y^2z^2 - 2(\lambda + w)e^{C(t)r+D(t)}x(t)y(t)z(t) + (\lambda + w)^2 e^{-\int_t^T\left(\psi(s)+bC(s)+\frac{a(y)}{\sqrt{\xi}}+g(z)\sqrt{\delta}\right)^2 ds} \quad (20)$$

With the expression of $A(t)$,$B(t)$,$C(t)$and $D(t)$ as 13-15

See the properties of $\hat{\pi}(t,x)$ in [10]

OPTIMAL SOLUTIONS AND EFFICIENT FRONTIER OF PROBLEM P(*w*)

According to the relationship between $PL1(\lambda,w)$ and $PL2(\lambda,w)$, if we define $\Gamma(\lambda)$ as the optimal objective function of $PL1(\lambda.w)$, then

$$\Gamma(\lambda) = P(0,r_0)x_0 - 2wQ(0,r_0)x_0 + w^2R(0) + \lambda^2(R(0) - 1) + 2\lambda(wR(0) - Q(0,r_0)x_0)$$

Noting that $R(0) = e^{-\int_t^T\left(\psi(s)+bC(s)+\frac{a(y)}{\sqrt{\xi}}+g(z)\sqrt{\delta}\right)^2 ds} < 1$, then $\lambda^* = arg\left(\max_{\lambda \epsilon R}\Gamma(\lambda)\right)$ exists and is

$$\lambda^* = \frac{wR(0) - Q(0,r_0)x_0}{1 - R(0)}$$

According to the relationship between $P(w)$ *and* $PL1(\lambda,w)$, the optimal strategy and efficient frontier of problem P(w) are concluded in theorem 5 below.

Theorem 5 for problem $P(w)$, the optimal strategy is

$$\pi(t, x(t), y(t), z(t)) = \frac{w - Q(0,r_0)x_0, y_0, z_0}{1 - R(0)} \frac{\left(bC(t) + \psi(t) + \frac{a(y)}{\sqrt{\xi}}P_1 + g(z)\sqrt{\delta}\right)}{\delta} - \frac{\left(bA(t) + \psi(t) + \frac{a(y)}{\sqrt{\xi}}PP_1 + g(z)\sqrt{\delta}\right)}{\sigma(t)} x(t)$$

And the efficient frontier is

$$Var(X(T)) = \frac{R(0)}{1-R(0)}[EX(T)] - \frac{Q(0,r_0)}{R(0)}x_0 \quad \text{Where} \quad R(0) = e^{-\int_t^T \left(\psi(s)+bC(s)+\frac{a(y)}{\sqrt{\xi}}+g(z)\sqrt{\delta}\right)^2 ds}$$

CONCLUSION

In this work, we carried out the continuous –time variance by using the Multiscale Stochastic volatility model. The assumption of multiscale stochastic volatility is more realistic but results in difficulties for handling the problem. Our wealth process corresponding to a candidate for the optimal strategy does not follow a homogeneous differential equation any more, which leads to more difficulties in deriving the analytic solution of the wealth process. But we have solved the problem successfully by constructing an auxiliary variable and deriving its analytic expression. On the other hand, for future works, results obtained in this paper can be applied or extended in Neutrosophic theory by applying Neutrosophic random variables (see [15,16,17]).

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